Task-related Factors that Influence the Spontaneous Use of Diagrams in Math Word Problems

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Summary: Diagrams are effective tools for problem solving. However, previous findings indicate that students generally do not use diagrams spontaneously. This study examined task-related factors that may influence the spontaneity of diagram use. Experiment 1 compared two possible explanations: the first, that the length-relatedness of the story context of the problem (i.e., whether it involves the measurement of length) determines the likelihood of diagram use; and the second, that the cognitive cost of transforming the situation described in the word problem to an abstract diagrammatic representation is the more important factor. Four math word problems, differing in their story context and structure, were administered to eighth-grade Japanese students (n=125) to solve. The results provide support for the cognitive transformation cost explanation. The results of experiment 2, in which the problems were administered to students in both Japan (n=291) and New Zealand (n=323), confirm this finding.

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INTRODUCTION
The importance of developing skills in diagram use

The development of students’ abilities in solving math word problems is important in mathematics education (see, e.g., Polya, 1945; Reed, 1999; Schoenfeld, 1985; see also the National Curriculum published by the Japanese Ministry of Education, 1998; the Standards published by the National Council of Teachers of Mathematics in the US, 2000). However, the development of such abilities is also considered as being particularly difficult for many students (Reed, 1999; Yoshida & Tajika, 1995) and, because of this, a variety of teaching approaches have been proposed to overcome those difficulties. One of the most effective of these is the cultivation of students’ abilities in using appropriate strategies and the development of associated knowledge about heuristics for solving word problems (Collins, Brown, & Newman, 1989; De Corte, Greer, & Verschaffel, 1996; Polya, 1945; Schoenfeld, 1985). Of the many kinds of heuristics and strategies that can be employed, the use of diagrams is considered the most effective for solving math problems (see meta-analysis conducted by Hembree, 1992).

Math teachers have been found to frequently use diagrams in teaching regular math classes (Dufour-Janvier, Bednarz, & Belanger, 1987) and, in other areas of problem solving research, the effectiveness of constructing diagrams has been empirically demonstrated (e.g. Cheng, 2002; Koedinger & Terao, 2002; Stern, Aprea, & Ebner, 2003; see also a review by Cox, 1999). Researchers working in the area of diagram use have discussed not only the beneficial effects that stem from their use (e.g. Ainsworth & Th Loizou, 2003; Cheng, 2004; Mayer, 2003; Pedone, Hummel, & Holyoak, 2001) but also the mechanisms by which they promote successful outcomes in problem solving (e.g. Larkin & Simon, 1987; Stenning & Oberlander, 1995; Zhang & Norman, 1994).

Students’ lack of spontaneity in using diagrams

Although self-constructed diagrams are potentially powerful tools for problem solving (see, e.g. Stern et al., 2003), problems have been identified in relation to students’ actual use of diagrams. These problems include inappropriate choice of diagrams for the problems being attempted, and failure to induce correct solutions even when appropriate diagrams have been constructed. However, perhaps the most serious and concerning of the problems—and the one addressed in this paper—is the lack of spontaneity with which students use diagrams when problem solving; in other words, students tend not to use diagrams of their own volition (Dufour-Janvier et al., 1987; Ichikawa, 1993, 2000; Uesaka, Manalo, & Ichikawa, 2007, 2010). This lack of spontaneity means that many students miss out on the efficacy that diagram use brings to problem solving—especially when one considers that, in most real-life problem solving situations, no diagrams are supplied, no hints are given for their use, and no instructors are present to encourage their use.

However, most of the research studies that have been carried out in this area have focused on the effects of diagram use and the mechanisms by which diagrams promote effective problem solving. Only a handful of studies have been undertaken to address the problems relating to their actual use. Uesaka et al. (2007) reported a study in which students’ perceptions about the efficacy of diagram use, their perceptions about the difficulties associated with using diagrams, and the extent to which they have internalized diagram use as part of their own repertoire of skills for problem solving were all found to affect the spontaneity with which students use diagrams. Uesaka et al. (2010) also demonstrated empirically in experimental classes that the acquisition of diagram drawing skills, combined with enhancement of students’ perceptions about the efficacy of diagram use, promotes the spontaneous use of diagrams. Furthermore, in order to enhance the quality of diagrams that students spontaneously produce, Uesaka and Manalo (2006) proposed and tested a classroom-based instruction method.
aimed at enhancing students’ conditional knowledge about diagrams. Consistent with this proposal, Novick (2006) argued that diagram literacy needs to be taught; she referred to evidence demonstrating that diagrammatic conventions as well as domain-specific knowledge are necessary for students to competently use diagrams.

The factors identified in the studies noted earlier were learner-related factors; the studies reported on individual perceptions and behaviors, knowledge, skills, and experiences that appeared to promote or inhibit the use of diagrams in problem solving. It is likely however that, apart from learner-related factors, there are also task-related factors that influence the use of diagrams. Identification of such factors is important, not least so that educators could develop effective methods for promoting the spontaneity with which diagrams are used in problem solving and take into account both student characteristics and features of the problems that are administered. However, the present authors are not aware of any previous studies that have investigated task-related factors that might influence the spontaneous use of diagrams in problem solving; this provided the impetus for the present study.

Length-relatedness in the story context of the math word problem

A task-related factor that has been proposed as affecting students’ spontaneity in using diagrams is the length-relatedness of the story context of math word problems (i.e. whether the problem is about distance or other measurement of length). Reed (1999, p. 82), for example referred to Hall, Kibler, Wenger, and Truxaw’s (1989) study and observed that ‘students used a diagram more often on the distance problems than on the less spatial word problems’. Although the purpose of the Hall et al. study was not to examine task-related factors that could influence diagram use, they did report inconsistencies in diagram use frequency across four types of word problems that they administered to their participants.

The length-relatedness explanation focuses on the story context attached to the math word problems. It is similar to an explanation provided for the ‘thematic-materials effect’, where an improvement in participant performance in problem solving appears to be ‘brought about by the use of “thematic” materials’ (Griggs, 1983, p. 18). More specifically, the explanation posits that the production of a correct response to a problem depends on the story context provided with the problem; the more ‘concrete’, ‘realistic’, or ‘thematic’ the story context, the more likely it is that a correct response or solution will be produced. This argument has been used in explaining variations in participant responses to problem solving tasks such as the Wason selection task (for reviews, see Evans, 1982; Griggs, 1983; Wason, 1983) and the Tower of Hanoi puzzle (see, e.g. Hayes & Simon, 1974).

There are some problems associated with using the length-relatedness explanation for variations in participant use of diagrams in problem solving. One of these problems relates to the Hall et al. (1989) study itself, because, as previously noted, the study was not aimed at identifying task-related factors that influence diagram use, there was not sufficient control employed in the study to confidently exclude the potential influence of other factors on participants’ use of diagrams. Thus, apart from differences in the story context provided with the problems, other factors could have had a bearing on the strategies that the participants used. Another important problem, which the length-relatedness explanation shares with the thematic-materials effect explanation, is that it provides only an explanation of the observed phenomena—but it does not explain the mechanisms that produce the phenomena. Thus, it is not clear why, according to this explanation, length-relatedness might promote the spontaneous use of diagrams.

In considering how it may be possible to overcome this important limitation of the length-relatedness hypothesis, research studies in the area of deductive reasoning provide some helpful ideas. To overcome the limitation of the thematic-materials effect explanation, in which it is not clear why more concrete or realistic story contexts promote better performance among participants, some researchers have focused on the cognitive processes that may drive the apparent influence of the superficial features of problems. For example, Cheng and Holyoak (1985) proposed that people have sets of reasoning schemas (that they referred to as ‘pragmatic reasoning schemas’) that comprise clusters of rules involving modals such as ‘permissions’ and ‘obligations’; according to Cheng and Holyoak, people can reason with greater efficacy if the problem they are dealing with can activate these schemas. Johnson-Laird’s (1983) theory of mental models is another attempt at explaining the underlying processes involved when human beings deal with problems. Johnson-Laird (1983) proposed that the number of mental models that people need to construct to solve a particular problem is a better gauge of its difficulty (i.e. compared with the more superficial features of problems). Like Cheng and Holyoak’s pragmatic reasoning schemas, Johnson-Laird’s mental models theory focuses on the cognitive processes involved in task execution rather than just the more superficial features of the tasks. As these earlier studies have demonstrated, more sophisticated explanations can be developed by considering the underlying human cognitive processes involved in problem solving. Thus, to better understand students’ decisions to use or not to use diagrams, a model or explanation that similarly deals with the underlying cognitive processes is required.

The cognitive cost of constructing diagrams

A possible alternative to the length-relatedness explanation is that the cognitive cost of constructing an appropriate diagram to solve the problem given is the more important determinant of whether a diagram will be used. Research studies in the area of learning strategy have also suggested that the cognitive cost relating to strategy use influences decisions that people make about whether to use a strategy or not. For example, McCombs (1988) model of how people decide to use a strategy suggests that people’s perception of the cost-efficiency of strategies affect their decisions of whether to use a particular strategy. Murayama (2003) and Sato (1998) also empirically demonstrated that if people
perceived a high cost in relation to a particular strategy, then the use of that learning strategy decreased. Using diagrams in problem solving also can be considered as a learning strategy; thus it is predicted that if the associated cognitive cost is low—and less mental effort is required—then a diagram will more likely be used, whereas if the cost is high, then a diagram will less likely be used.

To consider in more detail, the ‘cognitive costs’ involved in using diagrams to solve problems, it is helpful to envisage the procedural steps that are likely to be entailed. There are at least several steps involved from reading the word problem to correctly solving it with the use of an appropriate diagram. First, the student needs to carefully read the problem to understand its terms and requirements. During this process of understanding the problem, the student needs to mentally represent or visualize the concrete situation described in the problem, such as the component items and actions involved and their relationships to each other. At this stage, *if a diagram is to be used*, the student needs to translate the concrete situational representation to a diagrammatic representation. This diagrammatic representation, by necessity, needs to exclude irrelevant information that often comes with the word problems are presented, and distil only information that is directly relevant to solving the problem. From here, the student could then proceed to calculate the answer required.

Although there are cognitive costs associated with every step of this process, the cognitive cost involved in the translation step—from the concrete situational representation to the diagrammatic representation—is critical. This cognitive cost is mainly in terms of the degree of executive control required to complete the task (see, e.g. Miller & Cohen, 2001). In most cases, the student needs to translate the concrete situational representation to a more abstract one if the diagrammatic representation is to effectively portray the numerical values and relationships involved in the problem. More abstract representations include graphs, bar charts, or tables—the kinds of representation that Larkin and Simon (1987, p. 93) described as ‘artificial diagrams’, because they ‘do not describe an actual spatial arrangement’. During the translation step, various executive functions such as selective attention and decision making need to be utilized to a greater degree to effect the necessary arrangement and grouping of information that are to be used together in the resulting diagram (cf. Larkin & Simon, 1987). For example, irrelevant details from the problem such as what the items might look like need to be discarded in favor of structural details such as item categories and connections. The cognitive cost involved in such a translation could therefore be quite high and seemingly prohibitive for some students.

There are, however, math word problems that do not demand such high cognitive costs in the construction of an appropriate diagram for solving them. For such problems, a diagrammatic representation that resembles a pictorial representation of the concrete situation described in the problem is sufficient to facilitate the calculation of the correct answer. Such problems include those that describe ‘systems in real spaces’ (Larkin & Simon, 1987, p. 93), and those that require working out the total and different distances or lengths along a two-dimensional matrix (e.g. distances traveled by two people along a road, the comparative lengths of everyday objects). The translational step involved, from mentally representing the concrete situation of the problem to constructing a diagrammatic representation that portrays the numerical values involved and their relationships, is not very demanding. This is because the concrete situation described in the problem (e.g. two people starting out on opposite ends of a road) is exactly what needs to be depicted in the diagram (e.g. a line to represent the road and the locations of the two people). The actual translation is comparatively minimal, and thus the resulting cognitive cost is low. Many ‘length-related’ math word problems happen to fall into this category, and this may explain the apparent greater incidence of student diagram use when attempting to solve length-related problems.

The cognitive cost explanation posited here is congruent with other explanations that have been proposed about human reasoning, such as the pragmatic reasoning schema (Cheng & Holyoak, 1985) and the mental models theory (Johnson-Laird, 1983). Like these other explanations, the explanation that has been put forward here deals not only with the material or problem provided as the task but also the cognitive processes involved in engaging with that task.

**Overview and rationale of the present study**

To test the relative abilities of the cognitive cost and the length-relatedness explanations in predicting spontaneous diagram use, four problems were used (shown in Table 1). These problems differed on two dimensions: their length-relatedness (length versus non-length), and the structure of their solution (amount of work versus amount of change). On the length-relatedness dimension, problems either involved the measurement of length, or the measurement of weight or volume. On the structure-of-solution dimension, problems either focused on the amount of work that protagonists or objects carried out, or on the amount of change brought about by a protagonist or object.

As indicated in Table 1, the cognitive costs associated with translating the situations depicted in these problems to effective diagrammatic representations also varied. With the amount of work problems, the cost for the length problem is relatively low, whereas the cost for the non-length problem is relatively high. This is because, for the length problem, the cognitive cost of translating the concrete situation depicted in the problem (i.e. a road, two people walking towards a common place on the road) to an effective diagrammatic representation (e.g. a line to represent the road, the locations of the two people) is not very demanding. In contrast, for the non-length problem, translating the concrete situation depicted in the problem (i.e. a tank, two pipes from which water come out) to an effective diagrammatic representation (e.g. a bar chart or equivalent that depicts the quantities of water that came out from the two taps with the constraints given) would be more demanding. For this non-length problem, simply drawing a diagram that comprises a visual representation of the tank/container and the locations of the taps would be unhelpful towards arriving at the correct solution.

With the amount of change problems, the cognitive costs associated with translating the situations depicted in the
problems to effective diagrammatic representations did not differ: they were both high. This is because for the length problem, a diagram that simply comprises a line to represent the candle would be unhelpful towards arriving at the correct answer. Instead, a diagram in the form of a table or array of numbers to represent both the time and the corresponding lengths of the candle would be required—and this would be cognitively more demanding to construct. Likewise, for the non-length problem, drawing a diagram that simply represents the cheese and/or the mouse would be unhelpful. Again, a table or array of numbers to represent time and the corresponding amounts of cheese left would be considered as the kind of diagram that would be helpful in solving this problem—and the transformational cognitive cost associated with producing such a diagram would also be relatively high.

Thus, keeping in mind the assumptions of both the length-relatedness and the cognitive cost explanations, the experimental predictions formulated for the present study were as follows:

1. If the length-relatedness explanation were correct, observed spontaneous diagram use would be

   - In the amount of work problems: length problem > non-length problem
   - In the amount of change problems: length problem > non-length problem

2. If the cognitive cost explanation were correct, observed spontaneous diagram use would be

   - In the amount of work problems: length problem > non-length problem
   - In the amount of change problems: length problem = non-length problem

In other words, if the length-relatedness explanation were correct and only the story context linked to the problem matters, the advantage for the length problems should be observable in both amount of work and amount of change problems. However, if the cognitive cost explanation were correct, the advantage should not be observable in the amount of change problems as the cognitive cost of translating the length problem into a useful diagrammatic representation would be equivalent to that required for the non-length problem.

**EXPERIMENT 1**

The main purpose of this experiment was to carry out an initial test of the hypothesis: that the cognitive cost explanation would better predict students’ spontaneous use of diagrams compared with the length-relatedness explanation.

**Method**

The participants were 125 8th-grade students from Japanese public junior high schools, aged 13 to 14 years.

The dependent variable in this study was whether the participants spontaneously used diagrams in solving each of the problems they were given. There were two independent variables. The first of these was the story context, which was a between-subject factor. The second was the solution structure, which was the within-subject factor. The participants were randomly assigned to either the ‘length’ or the ‘non-length’ condition. This was done by randomly distributing equal numbers of booklets consisting of the two types of problems. In total, 63 of the students were assigned to the ‘length condition’, whereas the remaining 62 were assigned to the ‘non-length condition’.

As previously noted, four types of word problems, shown in Table 1, were used. Each participant received a booklet that contained two math word problems: one dealing with the amount of work that a protagonist or object carried out and the other on the amount of changed brought about by a protagonist or object. Whether these two problems had a length related or a non-length related story context depended on the condition that the participant was assigned to.

At the beginning of the experiment, the participants were informed that the purpose of the study was to understand the approaches that students use in solving math word problems.

### Table 1. Math word problems used in this study and the cognitive costs associated with their translation to effective diagrammatic representations (in brackets)

<table>
<thead>
<tr>
<th>Amount of work problems</th>
<th>Length</th>
<th>Non-length</th>
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<tbody>
<tr>
<td></td>
<td>Tom’s house and Hannah’s house are connected by one road, which is 600m long. Tom and Hannah talked on the telephone and decided to play with each other. They leave their own house at the same time and start to walk toward the other’s house. They meet on the road 5 minutes later. The place where they meet is 100m closer to Hannah’s house than the half-way point. How fast did Tom walk (per minute)?</td>
<td>There is a pond that can contain up to 600L of water. Now the pond is empty: so using two taps, A and B, you start to fill the pond with water. Water comes out of both taps at a constant rate. After 5 minutes, the pond is full. From tap A came 100L more water than half of all the water in the pond than half of all the water in the pond. How fast did water come out of tap A (per minute)? How fast did water come out of tap B (per minute)?</td>
</tr>
<tr>
<td>Amount of change problems</td>
<td>You light a candle and it starts to burn. It burns at a constant rate, which means that it burns at the same speed throughout. After 5 minutes, the candle is 10cm in length. After 7 minutes, it is 6cm. How long does it take for the candle to burn out?</td>
<td>A mouse starts to eat a piece of cheese. The mouse eats at a constant rate, which means it eats at the same speed throughout. After 5 minutes, there are 10g left of the cheese. After 7 minutes, there are 6g left. How long does it take for the mouse to finish eating the piece of cheese?</td>
</tr>
</tbody>
</table>

problems, and thus they were urged to show all of their work during the process of attempting to solve the problems. In the booklets given, the two problems were presented on separate pages (i.e. one problem to a page). Six minutes were given for working on each problem, and participants were asked not to proceed to the second problem without the experimenter’s instruction.

Results
The first author and a colleague independently examined each of the participants’ attempts at solving the problems and decided whether at least one diagram was used in their solution attempts. For the purposes of this study, a diagram was defined as any representation of the problem other than in the form of words, sentences, or numerical formulas. Thus, drawings counted as diagrams as did graphs and tables. Tables were defined as depictions of at least a pair of values in an array to represent the relationship between two variables. The inter-rater agreement (measured by calculating Cohen’s kappa coefficient) was found to be .98, which indicates almost perfect concordance. Examples of the diagrams produced by the participants are shown in Figure 1.

The definition of diagrams used in this research was congruent with definitions that have been used in previous diagrams research. Larkin and Simon (1987, p. 68), for example, considered ‘a data structure in which information is indexed by two-dimensional location’ as a diagrammatic representation and included in this category illustrations of pulleys as well as graphs in economics. Other authors have likewise included various forms of representations like tables, graphs, and illustrations of varying levels of abstraction in their conceptualization of diagrams (e.g. Blackwell & Engelhardt, 2002; Cox & Gravemeyer, 2003; Gravemeyer & Cox, 2004; Hegarty, Carpenter, & Just, 1991; Novick & Hurley, 2001). Hegarty et al., for example, used the categories of iconic, schematic, and charts and graphs to classify scientific diagrams. Their category of iconic diagrams included drawings and other forms of illustrations that depict the spatial relationships between the represented objects in an isomorphic manner.

The percentages of the participants’ spontaneous diagram use are shown in Figure 2. In the amount of work problems, the extent of spontaneous diagram use differed significantly for the length problem (72%) compared with the non-length problem (34%): $\chi^2(1) = 17.69, p < .01$. In contrast, in the amount of change problems, the difference between spontaneous diagram use in the length problem (46%) compared with the non-length problem (47%) was not significant: $\chi^2(1) = .01, n.s.$

Effect size independent of the sample size was calculated using the method suggested by Sugisawa (1999). This method uses the following formula where chi-squared tests are concerned (where $N$=the sample size):

$$\omega = \sqrt{\frac{\chi^2}{N}}$$

The effect size in the amount of work problem was .38, and in the amount of change problem, it was .03. Only the effect size of .38 in the amount of work problem falls between Cohen’s (1988) criteria for small and medium size effects.

Discussion
The results of Experiment 1 indicate that problems with length-related story contexts are not always associated with greater diagram use. Instead, the findings suggest that the cognitive cost explanation accounts for task-related differences in students’ spontaneous use of diagrams better than the length-relatedness explanation. These findings also contribute towards a better understanding of the possible reasons for the differences. In simple terms, the more straightforward it is to translate the concrete situation described in the problem to an appropriate diagrammatic representation, the more likely it is that a diagram will spontaneously be produced. Moreover, this same reason provides a viable

<table>
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<th>Length</th>
<th>Non-length</th>
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<tr>
<td>Amount of Work Problems</td>
<td><img src="image1.png" alt="Diagram" /></td>
</tr>
<tr>
<td>Amount of Change Problems</td>
<td><img src="image2.png" alt="Diagram" /></td>
</tr>
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Figure 1. Examples of diagrams produced by participants in solving the amount of work problems and the amount of change problems.
and ninety one of the students (mean age=13.28 years) were school students from Japan and New Zealand. Two hundred
The participants were 614 13- to 15-year-old secondary
Method
 tool
but also considers the use diagrams as a communication
ses the importance of teaching how to understand diagrams
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Figure 2. Percentages of spontaneous use of diagrams in Experiment 1

Figure 3. Percentages of spontaneous use of diagrams among Japanese participants in Experiment 2

explanation for why length-related problems appear to be more conducive to diagram use; it is because in many length-related problems, the translation required from the salient aspects of the problem to constructing a useful diagrammatic representation is less demanding. Drawing a simple picture of the salient aspects (e.g. the road, the target individuals along the road) may in fact already constitute a diagram that would facilitate working out the solution to the problem.

Because Experiment 1 was conducted using Japanese junior high school students as participants, and previous studies have reported that Japanese students as a group tend to be less inclined to use diagrams in math problem solving (Manalo & Uesaka, 2006; Uesaka et al., 2007) and are comparatively poor at using problem solving strategies (Organisation for Economic Co-operation and Development, 2004), a question remained as to the generalizability of the findings obtained. Would students, who may generally be better at using problem solving strategies (including diagrams), similarly exhibit differences in spontaneous diagram use as a function of the cognitive transformation cost demanded of the problems given? An answer to this question was sought in Experiment 2.

EXPERIMENT 2

As noted earlier, the purpose of Experiment 2 was to verify the findings of Experiment 1 with the use of participants who are reputed to demonstrate higher rates of spontaneous diagram use. Students in New Zealand were chosen for this comparison as they have previously been reported as spontaneously producing more diagrams when solving math word problems (Manalo & Uesaka, 2006; Uesaka et al., 2007). As Uesaka et al. (2007, p. 234) pointed out ‘the national curriculum in New Zealand not only stresses the importance of teaching how to understand diagrams but also considers the use diagrams as a communication tool’.

Method

The participants were 614 13- to 15-year-old secondary school students from Japan and New Zealand. Two hundred and ninety one of the students (mean age=13.28 years) were from Japanese schools, and 323 (mean age=13.97 years) were from New Zealand schools.

Data collection was carried out in each country, respectively. The design, materials, and procedures used were almost exactly the same as those in Experiment 1. The only exceptions were the amount of time given to the participants to solve each problem was reduced from 6 to 4 minutes, and the materials (i.e. the instructions and problems) were presented in the English language in New Zealand. The decision to reduce the amount of time was based on observations made during Experiment 1 that the majority of participants were able to complete the problems within 4 minutes. The wording equivalence of the Japanese and English instructions and questions were independently checked and confirmed by two of the authors’ colleagues who were fully fluent in both languages.

Results

The first author and a colleague independently scored the participants’ spontaneous use of diagrams using the same criteria employed in Experiment 1. The inter-rater agreement (measured by calculating Cohen’s kappa coefficient) was found to be .92, which again indicates almost perfect concordance. The raters discussed each instance where they differed to decide how to score those (i.e. used a diagram or not).

The results of the analyses carried out on the Japanese participants’ data confirmed the findings of Experiment 1. The percentages of spontaneous diagram use are shown in Figure 3. In the amount of work problems, there was a significantly higher proportion of participants who spontaneously used diagrams in the length problem (57%) compared with the non-length problem (26%): $\chi^2(1)=28.68, p<.01$. In contrast, with the amount of change problems, the proportion of participants who used diagrams in attempting to solve the length-related problem (38%) did not significantly differ from the proportion for the non-length related problem (31%): $\chi^2(1)=1.40, \text{n.s.}$

Effect size independent of the sample size was again calculated using the method suggested by Sugisawa (1999) noted earlier. The effect size in the amount of work problem was .31 and in the amount of change problem was .07. The effect size in the amount of work problem falls between
Cohen’s (1988) criteria for small and medium size effects. This therefore confirms the significant difference found in proportions of diagram use between the length and non-length ‘amount of work’ problems.

With the data obtained from the New Zealand participants, there was likewise a significantly higher proportion of students who used a diagram in attempting to solve the length ‘amount of work’ problem (59%) compared with the proportion for its non-length counterpart (16%): $\chi^2(1)=62.32, p<.01$. However, a significantly higher proportion of diagram users was also found for the length version (60%) compared with the non-length version (48%) of the amount of change problems: $\chi^2(1)=5.23, p<.05$. The relative sizes of these percentages are depicted in Figure 4.

Because significantly greater proportions of diagram use were found for both amount of work and amount of change length problems, it initially appeared that the New Zealand data supported the length-relatedness explanation. However, when effect sizes were again calculated, this revealed that although the effect size for the significant difference found in the amount of work problems ($\omega=.44$) falls between Cohen’s (1988) criteria for medium and large size effects, the effect size obtained for the significant difference in the amount of change problems ($\omega=.13$) failed to meet the criteria for small size effects. This therefore suggests that the significant proportional difference found in diagram use in the amount of change problems can simply be attributed to the big sample size used ($n=323$).

Thus, the findings of Experiment 2 confirm the findings of Experiment 1.

Discussion

The findings of Experiment 2 support the cognitive cost explanation for task-related differences in students’ spontaneous use of diagrams. With both Japanese and New Zealand students demonstrating higher spontaneous use of diagrams in length problems (as opposed to non-length problems) under the category of the ‘amount of work’ problems only, the evidence here points to the cost of cognitive translation as being the determining task-related factor in spontaneous diagram use.

General Discussion

The results of the two experiments described in this paper indicate that the cognitive cost explanation is more accurate in predicting the spontaneity of students’ diagram use in math word problem solving compared with the length-relatedness explanation. Although these explanations were contrasted in this paper, it is important to stress that they are by no means inconsistent with each other. In the study carried out by Hall et al. (1989), the factors associated with the cognitive cost explanation were confounded with those of the length-relatedness explanation, making it impossible to distinguish their respective influences on the spontaneity with which diagrams were used. This was part of the reason for seeking clarification of the relative capabilities of these explanations to predict diagram use. The findings obtained here can be interpreted as qualifying the length-relatedness explanation; students do spontaneously use more diagrams in solving length-related math word problems—but more specifically, those problems where the cognitive cost of constructing an appropriate diagram is not high. In other words, these are problems where a diagrammatic representation that resembles a pictorial representation of the concrete situation described in the problem would adequately facilitate the calculation of the correct answer. Many length-related math word problems happen to fall into this category, which likely explains the apparent greater incidence of student spontaneous diagram use when solving length-related problems. Therefore, the cognitive cost explanation proposed and tested here not only elaborates on the mechanisms involved in the length-relatedness explanation, it also clarifies the inherent parameters of the problems to which the length-relatedness explanation applies.

In the present study, the application of the cognitive cost explanation was tested on both students who are known to be generally lower in diagram use (the Japanese students) as well as students who are generally higher in diagram use (the New Zealand students). A survey carried out by Ichikawa, Seo, Kiyokawa, and Uesaka (2007), using COMPASS (Computational Assessment), has confirmed that diagram construction and use is a particular weakness among Japanese students; on the other hand, previous studies (Manalo & Uesaka, 2006; Uesaka et al., 2007) have reported comparably higher rates of diagram use among students in New Zealand. To have demonstrated the applicability of the cognitive cost explanation to both groups of students is important in that it indicates the findings were not merely an artifact of the educational context or environment that the students had been exposed to—therefore enhancing the generalizability of those findings.

Theoretical aspects of the findings

The findings of the present study contribute towards a better understanding of task-related factors that influence strategy use in problem solving. The authors acknowledge that there are potentially many reasons that could affect students’ spontaneity in using diagrams when problem solving—including, as mentioned earlier, diagrammatic conventions and domain-specific knowledge (Novick, 2006), perceptions about the
efficacy of diagram use, and skills in drawing diagrams (Uesaka et al., 2010). However, the few studies that have examined these reasons in particular have focused only on learner-related factors. The specific focus in the present study on the spontaneity with which students use diagrams in relation to the story context and structure of math word problems is therefore important because no previous research had been carried out on the task-related factors that may promote or deter diagram use—that is, apart from the long-standing and largely untested assumption that length-related story contexts promote diagram use (Hall et al., 1989; Reed, 1999). The alternative cognitive cost explanation that has been shown in this paper to be better at predicting diagram use is likely to also apply in other areas of strategy use and problem solving. Thus, examining the cognitive cost associated with different features of tasks/problems, and the generation of efficient heuristics for solving those problems, ought to be one priority in future research undertakings.

The difference between the explanations tested in this study (i.e. the length-relatedness and the cognitive cost explanations) is similar to the difference between competing theories of reasoning in problem solving. As noted earlier, whereas theories like the ‘thematic-materials effect’ (Griggs, 1983) emphasize the story context or the observable features that appear to influence the phenomenon in question, alternative theories like ‘pragmatic reasoning schemas’ (Cheng & Holyoak, 1985) and ‘mental models’ (Johnson-Laird, 1983) explain the same phenomenon but in terms of the underlying internal representations or cognitive processes. In effect, these cognitive process theories explain the very mechanisms that underpin the theories that focus only on the observable surface features. In the same vein, the cognitive cost explanation proposed in this paper provides one explanation (a task-based one) of the underlying cognitive mechanisms that influence the spontaneity of diagram use in problem solving. It also explains the cognitive mechanisms that make some length-related problems more likely to promote spontaneous diagram use.

The issue of cognitive cost in strategy use is one that has not been adequately explored. Although cognitive process models and theories have considered many aspects of internal representation such as class membership and permission schema, the matter of cost and its influence on strategy use has remained largely neglected. In other disciplines, a few studies have considered the cognitive cost issue; for example, in Lahlou’s (2000) notion of ‘cognitive attractiveness’, which relates to work behavior management, in Gronich’s (2006) ‘cognitive miser theory’, which attempts to explain seemingly illogical political decision making, and more recently in Kool, McGuire, Rosen, and Botvinick’s (2010) examination of how anticipated cognitive demand affects behavioral decision making. In light of the findings of the present study, further investigations appear warranted on the role and structural connections of cognitive cost in determining strategy choice and use in problem solving. Uesaka et al. (2007), for example, found that student perceptions of likely difficulties in diagram construction appeared to deter diagram use. Thus, in relation to this and the findings of the present study, it would appear useful to seek clarification of the interplay between human perceptions of cognitive cost (e.g. perception of difficulties that could be encountered in constructing a diagram) and task-inherent cognitive cost factors (e.g. the comparative number and difficulty of cognitive steps required to construct an appropriate diagram for a problem).

**Practical implications of the findings**

If, as the findings of this study suggest, cognitive cost associated with transforming the concrete terms of a problem into more abstract representations is a deterrent to the use of diagrams, finding viable solutions to make that cognitive cost more manageable appears imperative. The features of problem solving tasks will inevitably range from the ‘not-so-demanding’ to the ‘highly demanding’ in terms of the cognitive cost of constructing appropriate diagrams or other heuristics for solving them. Thus, solutions to the cognitive cost challenge will need to focus on the development of student capabilities in managing even the more demanding versions of such problems. Where diagram use is concerned, more training in their construction and uses would likely be helpful. However, other teaching strategies such as active comparison could additionally facilitate a better understanding of the relationships between salient task/problem-based features and their more abstract diagrammatic representations (see, e.g. Uesaka & Manalo, 2006). One objective of the instructions provided would need to be the reduction of students’ perceptions about the difficulties associated with diagram use (see McCombs, 1988; Uesaka et al., 2007) so that the cognitive cost of constructing more abstract diagrams would seem less prohibitive.

Future research in this area would also need to examine the cognitive organization and representation of diagrammatic knowledge so that the understanding gained can in turn be used for educational purposes such as in promoting the spontaneity of students’ diagram use in appropriate problem solving situations. There are many types of diagrams that can be constructed to help solve different types of problems, and those with sufficient expertise in problem solving (e.g. math teachers) can efficiently select the most appropriate kind in response to the problem given. Their knowledge about diagrams may be organized along at least two dimensions: one pertaining to the entity of diagrams (i.e. what diagrams are) and another relating to their purpose (i.e. what diagrams are for). Because different kinds and uses of diagrams (along these dimensions) are likely to be associated with variations in cognitive cost, such variations need to be considered and addressed in attempts at developing students’ skills in diagram construction and use. It may, for example, be possible to develop methods for reducing the higher cognitive costs that could be associated with the construction of more abstract diagrams by examining how such diagrams are internally represented and clustered with other information and details relating to problem solving. Such an examination could include considerations of the value and utility that students attribute to such diagrams.


